Eigenvalues and eigenvectors of the radiative transfer operator, and the spectral invariants in vegetation remote sensing

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Following up from the first Finnish Workshop on Radiative Transfer, we present the advances in the application and theory of the spectral invariants in vegetation remote sensing. The spectral invariants are based on the eigenvalues of the radiative transfer operator [1] and include the photon recollision probability, i.e., the probability that a photon which has interacted with a vegetation canopy, will interact withn the canopy again [2]. In recent years, the spectral invariants have found a number of applications in retrieving vegetation properties.

In addition to direct applications, the eigenvalue problem of the radiative transfer operator is importanabt for understanding the basics of vegetation-radiation interaction and allows effiient and robust methods to estimate the reflectance, transmittance and absorption of shortwave radiation in plant canopies. The eigenvalue problem for the general integro-differential radiative transfer equation is complex and diffcult to interpret in terms of the effects observed in the real world, e.g., in the field of passive optical remote sensing. Therefore, we analysed the eigenvalue problem in a simple the two-stream approximation, often applied in remote sensing of homogeneous plant canopies, with known analytical solutions. We derived the equation to calculate the eigenvalues for this important case and solved it numerically, as no analytical solution of the eigenvalue problem exists. We found up to an infinite number of eigenvalues and the respective eigenfunctions. The eigenfunctions are defined by a trigonometric or hyperbolic sine function for the downward radiation component, and by a sum of sine and cosine components for the upward direction. The eigenvalues are determined by the leaf area index, leaf reflectance to transmittance ratio and leaf orientation. Surprisingly, the first eigenvalue, related to the photon recollision probability, keeps changing even at large leaf area index values, where most other characteristics of the radiation field tend to saturate.

The results provide a simple approximation to calculate an important spectral invariant, photon recollision probability (p-value), related to the first eigenvalue, and the directionality of vegetation scattering. Eigenvalues and the p-value are mostly structural characteristics of vegetation with a minor dependence on optical properties of canopy elements.

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References

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