Inverse Days Conference 2022

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Book of Abstracts

Metal artifact correction in cone beam computed tomography using synthetic X-ray data

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Abstract

Metal artifact correction is a challenging problem in cone beam computed tomography (CBCT) scanning. Metal implants inserted into the anatomy cause severe artifacts in reconstructed images. Widely used inpainting-based metal artifact reduction (MAR) methods require segmentation of metal traces in the projections as a first step which is a challenging task. One approach is to use a deep learning method to segment metals in the projections. However, the success of deep learning methods is limited by the availability of realistic training data. It is challenging and time consuming to get reliable ground truth annotations due to unclear implant boundary and large number of projections. We propose to use X-ray simulations to generate synthetic metal segmentation training dataset from clinical CBCT scans. We compare the effect of simulations with different number of photons and also compare several training strategies to augment the available data. We compare our model's performance on real clinical scans with conventional threshold-based MAR and a recent deep learning method. We show that simulations with relatively small number of photons are suitable for the metal segmentation task and that training the deep learning model with full size and cropped projections together improves the robustness of the model. We show substantial improvement in the image quality affected by severe motion, voxel size under-sampling, and out-of-FOV metals. Our method can be easily implemented into the existing projection-based MAR pipeline to get improved image quality. This method can provide a novel paradigm to accurately segment metals in CBCT projections.

Dual modal imaging of two-phase flows using electromagnetic flow tomography and electrical tomography – state estimation approach

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Abstract

Accurate estimation of two-phase flow quantities such as phase fraction, velocity field, and volumetric flow rate of each phase is often required for, e.g., process control and product quality improvement. Recently, dual-modality imaging has attracted attention because single modality systems are often incapable of quantifying all relevant flow parameters. This paper discusses the development of a novel dual modal imaging system consisting of electromagnetic flow tomography (EMFT) and electrical tomography (ET) [1], and more specifically, the joint reconstruction of flow quantities based on these modalities using Bayesian state estimation [2].

State-space representation of the system consists of the observation models of EMFT and ET, and evolution models for the time-dependent state variables (here, the phase fraction distribution and velocity field). In EMFT, the multiphase fluid flowing in a pipe is exposed to an external magnetic field, which causes a Lorentz force on the electrically conductive fluid, and the resulting electric potentials are measured on electrodes attached on the surface of the pipe. By using the finite element method (FEM) for the approximation, the observation model for EMFT is in the form

$$U = H(\sigma(\phi))v_z + e_{v_z}, \tag{1}$$

where U is a vector consisting of electrode potentials, H is the forward operator, and ϕ is the spatially distributed phase fraction of the dispersed phase. Further, v_z is the axial component of the velocity field, and e_z is the additive observation noise [3]. In ET, we use the voltage-current (VC) system and the observation model is in the form

$$I = R(\sigma(\phi)) + e_{\sigma(\phi)}, \tag{2}$$

where R is the forward operator of ET and e_ϕ is observation noise.

Defining a new variable $\theta = [\phi^T \upsilon^T]^T$, we express Equations (1) and (2) in a concatenated form

$$\mathbf{y}_t = \mathbf{h}_t(\theta_t) + e_t, \tag{3}$$

where $\mathbf{y} = [U^T I^T]^T$ and $e = [e_v^T e_\phi^T]^T$ and t is a discrete time index. Furthermore, we model the time-dependencies of ϕ and υ_z by a stochastic convection-diffusion (CD) model [4] and a first-order Markov model, respectively. The FEM approximations of these surrogate models lead to an evolution model

$$\theta_{t+1} = f_t(\theta_t) + \omega_t \tag{4}$$

where f is the state transition operator and ω_t is the state noise process. Based on the state-space representation (Equations (3) and (4)), the state variable is estimated using extended Kalman filter (EKF) and the fixed-interval Kalman smoother (FIKS).

The performance of the proposed state estimation in dual-modality EMFT-ET is evaluated using a set of numerical simulations and further validated by experimental data. In these studies, the state estimates (especially FIKS) outperform the conventional stationary reconstructions in EMFT and ET. The proposed dual modal imaging system is expected to be applicable to industrial processes that involve for example oil-water flows.

References

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Sequential model correction for inverse problems

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Abstract

Inverse problems are in many cases solved with optimization techniques. With linear models and convex objective functionals first-order gradient methods are usually sufficient. With nonlinear models one must resort to second-order methods that are computationally more expensive. In this work we aim to approximate a nonlinear model with a linear one and correct the resulting approximation error. We develop a sequential method that iteratively solves a linear inverse problem and updates the approximation error. We analyze mathematically the convergence properties of the sequence and show numerically that the reconstruction results are superior to the conventional model correction method.

Adventures in Approximation Error

Simon Arridge

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Abstract

Jari Kaipio was one of the pioneers of Computational Bayesian Inverse Problems and put was key to putting Finland at the forefront of modern approaches to Inverse Problems. In this talk I will present one of his key ideas : Approximation Error Modelling. In this approach the concept is directly to account for systematic (epistemic) errors by modelling them through the statistics of a training set. The normal methods for Maximum A Posteriori estimation can be used, with a multivariate Gaussian term for the likelihood. The advantage is that low-cost computational models can be used for the forward and adjoint problems. In collaboration with Jari we studied this appraoch extensively for some non-linear problems in Optical Diffusion Tomography over a variety of systematic errors arising from various approximations, both computational and physical. In all cases the results gave results commensurate with more detailed models, at a fraction of the computational cost.

Field-free Line Magnetic Particle Imaging: Artifact Reduction for Time-dependent Tracer Concentrations

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Abstract

Magnetic particle imaging (MPI) is a promising still rather new tracer-based medical imaging technique, which does not depend on harmful radiation. Instead, magnetic material is injected into the patient's body being exposed to changing magnetic fields. Measuring the induced voltage signal resulting from the change in magnetization then allows to reconstruct the particle distribution within the body, hence enabling medical diagnostics. Different scanner implementations are available. We regard MPI using a field-free line (FFL) for spatial encoding. During data acquisition the FFL is steered through the field of view resulting in a scanning geometry resembling the one in computerized tomography (CT). Indeed, for static concentrations it has been shown that MPI data can be traced back to the Radon transform of the particle distribution. We extend this result towards time-dependent concentrations. Describing the corresponding dynamics using diffeomorphic motion models successfully applied in dynamic CT, we find that the MPI forward operator can be linked to an adapted version of the Radon transform. Finally, we jointly reconstruct particle concentration and adapted Radon data by means of total variation regularization for synthetic data.

Imaging water supply networks and vocal tracts

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Abstract

I will present some of my past and current work involving water supply networks, vocal tracts and other applications. The underlying mathematical model is the one dimensional wave equation and the inverse problem asks for the reconstruction of the wave speed or first order coefficient. What makes the problem interesting is that despite "being solved" in a general setting over 50 years ago there has not been much talk about implementations or collaboration with applied scientists. There are also interesting mathematical questions that still remain open, such as what is the simplest or lowest energy measurement that's required to determine vocal tract length or volume. Is is possible to determine these from passive measurements, like speech? Or is it required to send a controlled input wave and listen to the echo?

Determination of delta-shell and critically-singular potentials with local near-scattering data

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Abstract

We study the scattering problem with electric potentials. We consider potentials as a sum of *critically singular* and δ*-shell* components, whose direct problem can be solved using a mixture of Neumann series and Fredholm theory. We obtain an uniqueness result with *near-field* partial data for the corresponding inverse problem, by means of a Runge approximation and an interior regularity results.

Bayesian separation of small and large features for defect detection in CT of subsea pipes.

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Abstract

A non-destructive testing (NDT) application of X-ray computed tomography (CT) is defect detection in multi-layered subsea pipes in operation via 2D cross sectional scans. Detection of a defect and its location and size has associated uncertainties that are sought quantified to support decisions of whether or not to replace the pipe. To allow uncertainty quantification (UQ), we take a Bayesian approach to CT reconstruction and seek to determine the most informative prior. A pipe contains both large-scale structures (the pipe layers) and small-scale structures (defects). We have extensive knowledge of the pipe's internal layer structure and materials, which can be utilized to formulate a very informative prior. On the other hand, the prior must also allow deviations from the expected structure in order for the posterior to represent defects. This makes it challenging to formulate a single prior for the reconstruction.

In this work we propose to separate the reconstruction into a sum of two images; one representing expected, large features and one representing small, unknown features. Doing this allows us to impose a prior representing the well known pipe structure in one image and letting the other image contain any deviations from this. To express that only few and small defects are expected we employ a sparsity prior for the second image. A posterior is formulated w.r.t. both the large- and small-feature images and a Gibbs sampling approach is devised for exploring the posterior. An advantage of the proposed method is that defect detection is embedded in the reconstruction rather than being a post-processing step, which also simplifies UQ related to the defects. We demonstrate the method in numerical experiments and give an example of how to analyze the posterior samples to perform defect detection with UQ.

Uniqueness in an inverse problem of fractional elasticity

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Abstract

We study an inverse problem for fractional elasticity. In analogy to the classical problem of linear elasticity, we consider the unique recovery of the Lamé parameters associated to a linear, isotropic fractional elasticity operator from fractional Dirichlet-to-Neumann data. In our analysis we make use of a fractional matrix Schrödinger equation via a generalization of the so-called Liouville reduction, a technique classically used in the study of the scalar conductivity equation. We conclude that unique recovery is possible if the Lamé parameters agree and are constant in the exterior, and their Poisson ratios agree everywhere. This is a joint work with Professors Maarten de Hoop and Mikko Salo.

A one-step approach for retrieval and reconstruction of phase contrast X-ray data

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Abstract

Phase contrast X-ray imaging refers to the wide range of techniques which capture the changes in the phase of X-ray beams as they pass through an object. We focus on edge illumination as the method for acquiring X-ray phase contrast data. The edge illumination setup requires two masks, one in front of a sample and another at the detector. By moving the sample mask, one can vary the amount of illumination registered at the detector. These illumination changes form a Gaussian curve, the parameters of which can be retrieved as three information channels: attenuation, refraction and dark field. Refraction is the first order derivative of phase, so knowing refraction gives us the phase of the scanned sample. Normally, retrieval is performed prior to reconstruction in a two-step approach. In this work, we demonstrate a one-step approach which combines retrieval and reconstruction as a single model, and investigate appropriate subsampling schemes.

Reconstruction of the shape and spin-state model of potentially hazardous asteroid Apophis

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Abstract

Asteroid (99942) Apophis is a near-Earth and potentially hazardous asteroid in an excited rotation state. It will closely approach Earth in April 2029 at the minimum geocentric distance of 38,000 km. During this approach, the tumbling spin state of Apophis is expected to be altered by Earth's gravitation torque. The exact change depends on the orientation of Apophis during the close approach. Although the shape and spin-state model of Apophis was reconstructed from 2012/13 light curve observations by Pravec et al. (2014, Icarus 233, 48), the rotation parameters' precision was insufficient to predict the orientation for 2029.

We will present our analysis of new photometric observations of Apophis. By applying the light curve inversion technique of Kaasalainen (2001, A&A 376, 302), we reconstructed Apophis's spin state and shape. We aimed to invert both 2012/13 and 2020/21 data sets and reconstruct the spin state with high precision. The long interval of observations would enable us to precisely determine the rotation and precession periods and thus reliably predict the orientation of Apophis during its 2029 fly-by, compute the change of its spin state, and predict how the non-gravitational Yarkovsky effect will influence its post-encounter orbit, which is crucial for reliably compute the post-2029 impact probabilities.

Comparison of methods for fitting gaussian curves to edge-illumination data

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Abstract

Edge-illumination is a novel X-ray imaging method that is able to provide better contrast between biological tissues which have similar atomic numbers. It uncovers information on how the imaged material affects the phase of the radiation going through it. Additionally, attenuation and scattering information can be retrieved simultaneously, given a suitable imaging setup. The method works by using a coded aperture mask between an X-ray source and a sample to form beamlets. As the beamlets go through an object, the changes in phase also cause them to change direction. This causes sub-pixel shifts in the locations of the photons hitting the detector, which can be tracked. An important part of this tracking process is fitting a function to the measurements of photon distributions at each pixel. These functions are assumed to be Gaussian. In this work, we explore different ways of fitting Gaussians to X-ray edge-illumination data.

Stability estimates for the expected utility in Bayesian optimal experimental design

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Abstract

We study stability properties of the expected utility function in Bayesian optimal experimental design. We provide a framework for this problem in a non-parametric setting and prove a convergence rate of the expected utility with respect to a likelihood perturbation. This rate is uniform over the design space and its sharpness in the general setting is demonstrated by proving a lower bound in a special case. To make the problem more concrete we proceed by considering non-linear Bayesian inverse problems with Gaussian likelihood and prove that the assumptions set out for the general case are satisfied and regain the stability of the expected utility with respect to perturbations to the observation map. Theoretical convergence rates are demonstrated numerically in three different examples.

Local recovery of a piecewise constant anisotropic conductivity in EIT on domains with exposed corners

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Abstract

We study the local recovery of an unknown piecewise constant anisotropic conductivity in EIT (electric impedance tomography) on certain bounded Lipschitz domains Ω in $\mathbb{R}^2$ with corners. The measurement is conducted on a connected open subset of the boundary $\partial\Omega$ of Ω containing corners and is given as a localized Neumann-to-Dirichlet map. The above unknown conductivity is defined via a decomposition of Ω into polygonal cells. Specifically, we consider a parallelogram-based decomposition and a trapezoid-based decomposition. We assume that the decomposition is known, but the conductivity on each cell is unknown. We prove that the local recovery is almost surely true near a known piecewise constant anisotropic conductivity γ_0. We do so by proving that the injectivity of the Fréchet derivative $F'(\gamma_0)$ of the forward map F, say, at γ_0 is almost surely true. The proof presented, here, involves defining different classes of decompositions for γ_0 and a perturbation or contrast H in a proper way so that we can find in the interior of a cell for γ_0 exposed single or double corners of a cell of supp$H$ for the former decomposition and latter decomposition, respectively. Then, by adapting the usual proof near such corners, we establish the aforementioned injectivity.

This is based on joint work [1] with Maarten V. de Hoop, Ching-Lung Lin, Gen Nakamura, and Manmohan Vashisth.

References

[1] M. V. de Hoop, T. Furuya, C. L. Lin, G. Nakamura, M. Vashisth, "Local recovery of a piecewise constant anisotropic conductivity in EIT on domains with exposed corners", arXiv:2202.06739, (2022).

Learned reconstruction methods with convergence guarantees

Andreas Hauptmann

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Abstract

In recent years, deep learning has achieved remarkable empirical success for image reconstruction. This has catalyzed an ongoing quest for precise characterization of correctness and reliability of data-driven methods in critical use-cases, for instance in medical imaging. Notwithstanding the excellent performance and efficacy of deep learning-based methods, concerns have been raised regarding their stability, or lack thereof, with serious practical implications. Significant advances have been made in recent years to unravel the inner workings of data-driven image recovery methods, challenging their widely perceived black-box nature.

In this short talk I will outline relevant notions of convergence notions for data-driven image reconstructions and how present a high-level view on how these can be achieved in practice.

This talk is based on a survey paper with Subhadip Mukherjee, Ozan Öktem, Marcelo Pereyra, and Carola-Bibiane Schönlieb: *Learned reconstruction methods with convergence guarantees* (to appear in IEEE Signal Processing Magazine).

Remote controlled phantom for dynamic tomography

Tommi Heikkilä

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Abstract

Tomography is a well known imaging modality widely used in different applications, which has also motivated a lot of inverse problems research which often try to push the boundaries of what is possible. A good example is dynamic tomography where the object of interest changes during the tomographic measurement process causing reconstruction errors and motion artifacts if traditional methods are used. In recent years we have seen active research and many promising results on dynamic tomography. However openly available real data for assessing and comparing the quality of novel methods has not advanced as fast.

The Spatio-TEmporal Motor-POwered (STEMPO) phantom is a computer controlled mechanical device designed for collecting dynamic X-ray tomography data using various measurement setups. Properties of both the device and the currently available data are briefly described and to illustrate its capabilities, some examples are given using methods such as sparse + low-rank matrix decomposition and spatio-temporal wavelet regularization.

Diffuse optical imaging of the preterm neonate brain with a full-term neonate atlas model

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Abstract

Diffuse optical tomography (DOT) is a relatively convenient method for imaging the hemodynamic changes related to neuronal activity or injury in the brain cortex. Newborns can be imaged at bedside or while laying on their parent's lap, and the smaller brain and the thinner extracerebral layers increase the penetration of light into the brain.

The inverse problem of DOT image reconstruction is often solved in an atlas model of the head anatomy, meaning that the subject's head shape and sensor locations are combined with the inner anatomy from another individual or a population-level average. In newborns, the importance of the correspondence of the gestational ages of the imaged subject and the atlas model is highlighted due to the rapid brain growth and maturation during the last trimester of pregnancy.

In this talk, we observe how the usage of a full-term individual's atlas model affects the reconstruction of an activity simulated in a very preterm neonate's model. We use the Monte Carlo method as the forward solver, and present our approach for image reconstruction from frequency-domain amplitude and phase shift measurements.

Interface Estimation in a Three-Phase Separator using a Tomographic Profiler

Marzieh, Hosseini (a), Antti, Nissinen (a), Jonathan , Vänska (a), Arto , Voutilainen (a), Pekka , Kaunisto (a), Johan , Sjöblom (a), Mika , Tienhaara (a)

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Abstract

Information on interface levels is an indispensable part of maintaining and improving the performance of various separation processes. Non-optimal levels or unawareness of the process state can result in several problems, such as issues in wastewater processing, poor product quality, or unnecessarily low throughput. Electrical tomography is a rapidly emerging technology for level measurements in separation processes. It is well-suited for measuring various interface levels in challenging operating conditions where good accuracy and decent time resolution are needed. Rocsole Ltd has been developing tomography-based level measurement technology, and this paper summarizes the results from different test cases carried out in a controlled test environment.

The test environment is created based on the conditions found in an enhanced oil recovery process. During the tests, different levels of liquids with various properties are used, and their electrical properties are examined using a tomographic probe sensor. The probe sensor sends an excitation voltage to an electrode ring, and the current response is measured on other electrode rings. The liquid interfaces and their properties can be characterized using the collected measurements.

The performance of the probe sensor is evaluated in a simulated separator process condition by installing it in a 300l test vessel and adding transmission oil, salt water, and 20%, 40%, and 60% water-cut emulsions. High salinities were used in both the water and emulsions since seawater injection is commonly used as part of an Enhance Oil Recovery (EOR) process. The error in estimated water-emulsion and oil-gas interfaces was within $\pm$ 1 cm (less than 1% deviation from actual values). Some mixing occurs as various fluids are inserted into the test vessel. The mixing of emulsion and water was relatively minor. However, oil and emulsion were mixed significantly, and no clear interface could not be identified. The estimated conductivity profile shows the conductivity gradient in the emulsion-oil interface correctly. The estimated conductivity values for

Phase-contrast THz-CT for non-destructive testing

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(a): Johann Radon Institute, Linz, Austria, (b): Industrial Mathematics Institute, Johannes Kepler University, Linz, Austria

Abstract

In this talk, we consider the imaging problem of THz computed tomography (THz-CT), in particular for the non-destructive testing of extruded plastic profiles. We derive a general nonlinear mathematical model describing a full THz-CT experiment, and consider several approximations connecting THz-CT with standard computed tomography and the Radon transform. The derived models are based on geometrical optics, and contain both the THz signal amplitude and phase. We consider several reconstruction approaches using the corresponding phase-contrast sinograms, and compare them both qualitiatively and quantitatively on experimental data obtained from 3D-printed plastic profiles which were scanned with a THz time-domain spectrometer in transmission geometry.

Edge-promoting sequential Bayesian experimental design for X-ray imaging

Nuutti Hyvönen

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Abstract

This work considers sequential edge-promoting Bayesian optimal experimental design for X-ray imaging. The process of computing a total variation type reconstruction for the absorption inside the imaged body via lagged diffusivity iteration is interpreted in the Bayesian framework. Assuming a Gaussian additive noise model, this leads to an approximate Gaussian posterior with a covariance structure that contains information on the location of edges in the posterior mean. The next projection geometry is then chosen through A- or D-optimal Bayesian experimental design, which corresponds to minimizing the trace or the determinant of the updated posterior covariance that accounts for the new projection. The method is tested via numerical experiments based on simulated measurements.

Geophysics and algebraic geometry

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Abstract

Many areas of interest within the Earth are anisotropic, meaning that the speed of sound is different in different directions. It turns out that pressure waves are far better behaved than shear waves, but fortunately the different polarizations are coupled together through algebraic geometry. I will explain the surprising power of algebraic geometry in the study of anisotropic inverse problems.

Mumford-Shah regularization in electrical impedance tomography with complete electrode model

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Abstract

In electrical impedance tomography, we aim to solve the conductivity within a target body through electrical measurements made on the surface of the target. This inverse conductivity problem is severely ill-posed, especially in real applications with only partial boundary data available. Thus regularization has to be introduced. The Mumford–Shah regularizer familiar for image segmentation is appropriate for targets consisting of several distinct objects or materials. It is, however, numerically challenging.

In this talk, we discuss some theoretical and numerical aspects of the Ambrosio-Tortorelli approximation of the Mumford-Shah regularizer combined with the Complete electrode model (CEM).

A first-order optimization method with simultaneous adaptive pde constraint solver

Bjørn Jensen (a), joint work with Tuomo Valkonen (b)

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(b): Department of Mathematics and Statistics, University of Helsinki, Finland

Abstract

We consider a pde-constrained optimization problem and based on the nonlinear primal dual proximal splitting method, a nonconvex generalization of the well-known Chambolle-Pock algorithm, we develop a new iterative algorithmic approach to the problem by splitting the inner problem of solving the pde in each step over the outer iterations. In our work we split our pde-problem in a fashion similar to the classical Gauss-Seidel and Jacobi methods, though other iterative schemes may be fruitful too. We show through numerical experiments that significant speed ups can be attained compared to a naïve full pde-solve in each step, and we prove convergence under sufficients second-order growth conditions.

Quasi-Monte Carlo for Bayesian optimal experimental design

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Abstract

Contemporary quasi-Monte Carlo (QMC) methods are based on tailoring specially designed cubature rules for high-dimensional integration problems. By leveraging the smoothness and anisotropy of an integrand, it is possible to achieve faster-than-Monte Carlo convergence rates. To this end, QMC methods have become a popular tool for numerical treatment of partial differential equations involving random coefficients. Meanwhile, the goal in Bayesian optimal experimental design is to maximize the expected information gain for the reconstruction of unknown quantities when there is a limited budget for collecting measurement data. In this talk, we derive tailored QMC cubature rules to alleviate the computational burden associated with Bayesian optimal experimental design problems governed by partial differential equations.

Inverse Problem called the Department of Applied Physics

Tero, Karjalainen

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Abstract

In Eastern Finland we have this special inverse problem called the Department of Applied Physics. This special problem belongs to the application area of system identification. This time evolving system is a non-linear with some stochastic characteristics. There are some discrepancies in identifying model parameters and observations. Crude arbitrariness will be used in defining initial values. The set of observations at different time steps will be presented and some boundary conditions are highlighted.

The presentation will go through some major events in the history of problem definition and evolution. To the best of our knowledge this is the first attempt to also formulate a model for this ill-posed problem.

Estimating the singular support of the limited-angle tomographic reconstruction using persistent homology

Elli Karvonen

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Abstract

In some applications, we can only use limited-angle X-ray tomography, which results in a much harder reconstruction problem than a full-angle case. Despite an algorithm, the singularities, i.e., the boundaries of the target object, cannot be detected stably. This means parts of the boundaries are missing. If we suppose that the target object has disjoint regions, the boundaries of those regions form cycles. Thus we can try to find missing parts of these cycles (boundaries) with help of homology.

The complex wavelets provide a computational method for finding stable singularities and dividing them into six sets based on their directions. Using prior information about singularities' directions, it is possible to estimate unknown singularities based on known singularities. Persistent homology can identify when known singularities and estimated singularities occur together and form cycles. This way singular support estimation can be reduced. Simultaneously one can get information about the number of regions.

Conductivity reconstruction towards higher genus surfaces

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Abstract

We study the problem of recovering an electrical conductivity from interior power density measurements on a two-dimensional Riemannian manifold. This problem arises in Acousto-Electric Tomography and is motivated by the geometric Calderón problem of recovering the metric from the Dirichlet-to-Neumann map. In contrast to the geometric Calderón problem, we consider a conductive Riemannian manifold and treat the conductivity and metric separately. Assuming that the metric is known, for two-dimensional Riemannian manifolds with genus zero, we highlight in this talk that under certain assumptions on the power density data it is possible to recover the conductivity uniquely and constructively from the data. Furthermore, we address the same problem on a two-dimensional Riemannian manifold with higher genus.

A Bayesian approach for inclusion detection using parametrisations

Kim Knudsen (a), Aksel Rasmussen (a)

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Abstract

In this talk, we consider a Bayesian approach of recovering inclusions in PDE-based inverse problems using parameterisations. Examples include the level-set parametrisation which constructs a piecewise constant function from the level sets of a smooth function. We will consider this approach from the perspective of Bayesian consistency. This framework ultimately gives conditions for the parametrisation, forward map and prior such that the posterior mean converges to the ground truth in probability when noise on the data goes to zero. We then test the level-set method for two non-linear and mildly ill-posed PDE-based inverse problems that mimic a setting where these conditions are satisfied.

Reconstruction of an impenetrable obstacle in anisotropic inhomogeneous background

Pu-Zhao Kow (a), Jenn-Nan Wang (b)

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(b): Institute of Applied Mathematical Sciences, NCTS, National Taiwan University, Taipei, Taiwan.

Abstract

We are interested in the problem of determining the shape of (sound-soft or impedance) obstacles in the acoustic wave scattering from the knowledge of the (eigenvalues and eigenfunctions) far-field operator. The main idea is to choose suitable incident field according to the (known) anisotropic inhomogeneous background. We usually choose plane wave (which verifies Helmholtz equation) as incident field when the background is isotropic homogeneous. However, some analogue of well-known properties of the far-field pattern of scattered field may not hold with such choice of incident field, for example, the "first reciprocity relation".

Some part of this talk is is related to my work [IMA J. Appl. Math. **86** (2021), no. 2, 320–348] (there is a corrigendum).

Disentangling vegetation fluorescence signal from space-based measurements of reflected solar radiation

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(a): Finnish Meteorological Institute, Kuopio, Finland, (b): Department of Applied Physics, University of Eastern Finland, Kuopio, Finland, (c): Finnish Meteorological Institute, Helsinki, Finland

Abstract

Photosynthesis is one of the most important mechanisms that enable life on Earth. It is a process where sunlight is converted to chemical energy by synthesizing sugars using water from the soil and carbon dioxide from the air. Thus understanding this mechanism is crucial to have a better grasp of our climate system and the Earth's carbon cycle.

One commonly used proxy value to measure photosynthetic activity of plants is solar-induced fluorescence (SIF), which is a subtle light emission signal emitted around the red and the near-infrared wavelengths of the electromagnetic spectrum.

Current methods to retrieve SIF with satellite remote sensing are statistically-based and usually utilize solar Fraunhofer lines to disentangle the SIF signal from the atmospheric and surface contribution of the satellite measured radiance. This disentanglement process is a highly ill-posed problem due to many absorption and scattering events present in the atmosphere and since the SIF signal is relatively small.

In this work, we investigate the SIF retrieval problem using simulated datasets. The simulated satellite instrument is the TROPOspheric Monitoring Instrument (TROPOMI) on board the Copernicus Sentinel-5 satellite. TROPOMI has a good spectral resolution making it a suitable candidate for accurate global SIF retrievals. The results of this study are used to develop a physically-based SIF retrieval with real TROPOMI measured data.

Recovery of singularities from fixed angle scattering data for biharmonic operator in dimensions two and three

Jaakko Kultima

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University of Oulu, Oulu, Finland

Abstract

The inverse fixed angle problem for operator $\Delta^2 u + V(x, |u|)u$ is considered in dimensions $n = 2, 3$. We prove that the difference between the inverse fixed angle Born approximation and the function $V(\cdot, 1)$ is smoother than the function V itself in some Sobolev scale. This allows us to deduce that the main singularities of the perturbation V can be reconstructed from the knowledge of the scattering amplitude with some fixed incident angle.

Using multi-frequency electrical impedance tomography (MFEIT) without phase data to reconstruct complex conductivity distribution

Petri, Kuusela (a)

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Abstract

Electrical impedance tomography (EIT) is an imaging modality, where the conductivity distribution inside imaging domain is reconstructed based on electrical measurements made on the boundary. Traditionally, the only way to reconstruct the complex conductivity (in practice, conductivity and permittivity) has been to measure complex signals (i.e. both magnitude and phase). We present a novel multi frequency EIT (MFEIT) method, which enables reconstructing the complex conductivity based on only magnitude measurements on the boundary. The lack of phase data is compensated for by having measurements at multiple frequencies. Both simulated and experimental studies of the novel method are presented.

Coupled finite element and ultra-weak variational formulation for Maxwell equations

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(b): HyperComp Inc., Los Angeles, USA [file://(c)]
(c): Department of Mathematical Sciences, University of Delaware, Newark, USA

Abstract

The Ultra-Weak Variational Formulation (UWVF) provides an efficient framework for simulating electromagnetic wave fields in challenging computational geometries. In this work, the method is applied to situations that contain different levels of geometric components. In the applications considered, these components include, for example, an antenna and the main body where it is mounted and that scatters the wave field. The antenna model often leads to geometry that contains sharp corners, that in turn can be problematic for the plane wave basis used in the UWVF. In this work, the simulation model is coupled to polynomial basis functions, that aim to improve the accuracy on those elements located near the sharp corners. In addition, to relax the element size requirement on the scattering object or interface between different materials, support for curved element types has been studied. The applicability of the simulation software developed in this project is examined using numerical examples.

Mapping properties of neural networks and inverse problems

Matti Lassas

University of Helsinki

Abstract

We will consider mapping properties of neural networks, in particular, injectivity of neural networks, universal approximation property of neural networks and the properties which the ranges of neural networks need to have. Also, we study approximation of probability measures using neural networks composed of invertible flows and injective layer and applications of these results in inverse problems.

Recent R & D Topics in Mammography and CBCT Imaging

Mikko, Lilja

Software and Algorithms, Planmed Oy, Helsinki, Finland

Abstract

In medical imaging R&D on image reconstruction and processing methods, the typical KPIs include computational efficiency; baseline image quality (in terms of image contrast, sharpness and noise); robustness and generalizability in terms of input data; and suppression of artefacts emanating from either the image acquisition and reconstruction process or post-processing operations applied to the "raw" images. Moreover, the outcome of the work should have a positive business impact, which often means that the contemporary R&D topics are driven by customer demand and competitor analysis.

Recent R&D work at Planmed and Planmeca is presented for mammography reconstruction (tomosynthesis, synthetic mammogram and contrast-enhanced mammogram), mammography and dental X-ray image enhancement, and CBCT motion artefact suppression as well as image stitching.

The anisotropic Calderón problem at large fixed frequency on manifolds with invertible ray transform

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Abstract

We consider an inverse problem of recovering a potential from the Dirichlet to Neumann map at a large fixed frequency on certain Riemannian manifolds. Here we extend the previous work of G. Uhlmann and Y. Wang [UW21] to the case of simple manifolds, and more generally to manifolds, where the geodesic ray transform is stably invertible.

References

[UW21] G. Uhlmann and Y. Wang, *The anisotropic calderón problem for high fixed frequency*, arXiv, 2021. arXiv:2104.03477.

Sparsity promoting reconstructions via hierarchical prior models in diffuse optical tomography

Anssi Manninen (a)

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Abstract

Diffuse optical tomography (DOT) is a severely ill-posed nonlinear inverse problem that seeks to estimate optical parameters from boundary measurements. In the Bayesian framework, the ill-posedness is diminished by incorporating *a priori* information of the optical parameters via the prior distribution. In case the target is sparse or sharp-edged, the common choice as the prior model are non-differentiable total variation and ℓ^1 priors. Alternatively, one can hierarchically extend the variances of a Gaussian prior to obtain differentiable sparsity promoting priors. By doing this, the variances are treated as unknowns allowing the estimation to locate the discontinuities.

Previous work with hierarchical models was focused on linear inverse problems. In this work we extended the use of hierarchical models to the nonlinear inverse problem of DOT. To compute the MAP estimates, a previously proposed alternating algorithm was adapted to work with the nonlinear model. The performance of the hyperpriors was evaluated in numerical simulations, demonstrating their ability to promote reconstructions of sparse and sharp-edged targets.

Experimental and Computational Resources for Computed Tomography Research at the University of Helsinki

Alexander Meaney, Samuli Siltanen (a)

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Abstract

Computed tomography is an imaging modality in which X-ray projections taken from different directions around an object are combined to form cross-sectional images by using a reconstruction algorithm [1]. Significant theoretical and applied research in these algorithms has been conducted in the FIPS research network. However, there still remains a significant gap between this basic research and putting the algorithms into practical use with real data and applications.

This presentation will give an overview of the experimental and computational resources developed at the University of Helsinki to bridge this gap. Experimental resources include a custom-built cone beam computed tomography scanner specifically designed for collecting datasets for the development of CT algorithms. We can conduct measurements using either a conventional energy-integrating X-ray detector or a photon counting detector. Computational tools include open datasets and HelTomo [2], a Matlab toolbox specifically designed to be used with X-ray data measured our laboratory. It provides an easy yet flexible user interface, designed for researchers to be able to focus on the essential: computing both 2D and 3D CT reconstructions and developing new reconstruction algorithms.

References

[1] Jiang Hsieh and Thomas Flohr. Computed tomography recent history and future perspectives. *Journal of Medical Imaging*, 8(5):052109, 2021.

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How to Quantify Model Error

Puyuan, Mi

Compute, DTU, Lyngby, Denmark

Abstract

To solve an inverse problem, we need to set up some numerical models. It is always the case that relatively ideal methimetical models would lead to nice reconstructions but high computational cost, while easily implementable models would lead to fast reconstructions but poor accuracy. In this project, we try to figure out how much influence would the model error introduced by applying numerically implementable model cause, and how to reduce the influence. By analysing the model error from a Bayesian prospective, we give illustration under UQ framework.

The effect of aerosols for carbon dioxide emission plume retrieval

Antti Mikkonen (a), Hannakaisa Lindqvist (a), Johanna Tamminen (a)

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Abstract

Greenhouse gas emissions from anthropogenic activities is the main driver of current global climate change. Emission monitoring is essential in aiding the emission reduction effort and a feasible way for attaining global coverage are satellite observations. Recent developments in space-based hyperspectral cameras open up new possibilities for greenhouse gas emission monitoring also on a smaller scale.

Most of the anthropogenic greenhouse gas emissions originate from urban areas. Urban areas are also sources of atmospheric aerosols, which decrease the local air quality and complicate the atmospheric radiative transfer. Even slight concentrations of atmospheric aerosols cause considerable inaccuracies in space-based remote sensing observations of carbon dioxide (CO_2).

In this work, a simulated hyperspectral camera scene of a co-emitted CO_2 and aerosol plumes is examined. The non-linear inverse problem of determining the aerosol and CO_2 content of the atmosphere in each camera pixel is studied. The radiative coupling of adjacent camera pixels is dependent on the viewing geometry and it can be used to gain extra information for the retrieval process. This method is not only applicable in recent and upcoming satellite missions such as PRISMA or CO2Image, but also for airborne and drone-based hyperspectral observations.

The Time Domain Linear Sampling Method

Peter Monk (a)

(a): Mathematical Sciences, University of Delaware

Abstract

In the frequency domain, the linear sampling method (LSM) of Colton and Kirsch offers a simple way to determine the boundary of a scatterer provided sufficient multi-static data is available. However, for limited aperture data, the LSM may yield poor reconstructions. In an attempt to reduce the number of transmitters and receivers it has been suggested to use an analogue of the LSM in the time domain (the TD-LSM). In this talk I will outline the TD-LSM and the theoretical background underlying the method. I shall also describe the resulting numerical algorithm. Numerical results for the Helmholtz equation show some promise towards the desired decrease in sources and receivers. Finally I shall mention some open problems.

Model-based reinforcement learning and inverse problems in extreme adaptive optics control

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(a): Lappeenranta–Lahti University of Technology
(b): European Southern Observatory

Abstract

The field of exoplanet research is one of the most rapidly expanding research fields in modern astrophysics. In recent decades, astronomers have found most exoplanets via indirect techniques such as the transit and radial velocity method. The direct imaging technique called high contrast imaging (HCI) enables new ways to expand our knowledge of these exoplanets and exoplanetary systems. However, direct imaging of exoplanets is challenging due to the high contrast ratio and small angular separation from the host star. Thus, HCI detections, so far, are mostly limited to a few tens of young and luminous giant exoplanets.

The new generation of HCI instruments, under development, will push direct imaging to increasingly challenging areas, discovering and characterizing exoplanets dimmer and closer to their host start. The ultimate goal is direct imaging and characterization of potentially habitable exoplanets. On ground-based telescopes, HCI instruments are equipped with eXtreme Adaptive Optics (XAO) that correct the phase fluctuations caused by the atmosphere. With an optimized instrument design, the residuals left by XAO correction set the limitation of sensitivity; thus, minimizing the XAO residuals is a crucial objective for ground-based HCI. Further, most habitable exoplanets are located at small angular separations from their host stars, where current XAO control algorithms leave strong residuals of stellar light that could be suppressed with more advanced algorithms.

We explore novel data-driven control methods for XAO control that cope with crucial limitations of traditional control laws, such as temporal delay and calibration errors. Improvement in these potentially reduce the residual flux of stellar light in the coronagraphic point spread function and thus enables fainter observations closer to the host star.

Model corrected primal-dual deep equilibrium models for photoacoustic tomography

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Abstract

Photoacoustic tomography is a hybrid imaging modality that combines optical contrast with ultrasonic detection, and it has shown the potential in a variety of biomedical applications. In these applications, real-time imaging is usually in demand and fast reconstruction algorithms are required. Learned iterative reconstruction shows great promise for acceleration of tomographic reconstructions and shows empirical robustness to model perturbations. Nevertheless, an adoption for photoacoustic tomography is hindered by the need to repeatedly evaluate the computational expensive forward model. Computationally feasibility can be obtained by the use of fast approximate models, but a need to compensate model errors arises. We address this in a primal-dual framework, where a model correction is learned in data space and a learned proximal operator in image space. Following the successful deep equilibrium models, we train a fixed point iteration both in data and image space to obtain robust and convergent reconstructions. Results are presented for two-dimensional simulations and in-vivo measurements in full three dimensions.

Injective Machine Learning Architectures for Inverse Problems

Michael Puthawala

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Abstract

Machine Learning has emerged as a powerful and diverse tool for solving a range of problems in science and applied mathematics. In the context of inverse problems, machine learning has, for example, been used in place of an inverse solver or as a regularizer to an existing method. In this talk we investigate the question of when neural network architectures are themselves injective. We find that networks formed by combining injective ReLU layers with bijective flow networks, we can build networks that are provably end-to-end injective. Additionally, we find that such networks come with a bevy of built-in benefits that make them well-suited for application. These include inverse stability which enables Bayesian uncertainty quantification, guarantees on universality, a novel layerwise invertibility result. Finally, we conclude with a new application where such networks are combined with a coordinate projection to produce a network that is a universal approximator of maps between compact smooth submanifolds that are locally bilipschitz, but can globally be quite complex.

New insight to EIT reconstruction using virtual X-rays

Siiri Rautio

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Abstract

The mathematical model of electrical impedance tomography (EIT) is the inverse conductivity problem introduced by Calderón. The aim is to recover the conductivity σ from the knowledge of the Dirichlet-to-Neumann map Λ_σ. It is a nonlinear and ill-posed inverse problem.

We introduce a new reconstruction algorithm for EIT, which provides a connection between EIT and traditional X-ray tomography. We divide the exponentially ill-posed and nonlinear inverse problem of EIT into separate steps. We start by mathematically calculating so-called virtual X-ray projection data from the DN map. Then, we perform explicit algebraic operations and one-dimensional integration, ending up with a blurry Radon sinogram. We use neural networks to deconvolve the sinogram and finally, we can compute a reconstruction of the conductivity using the inverse Radon transform. We demonstrate the method with simulated data examples.

This is a joint work with Samuli Siltanen, Matti Lassas, Rashmi Murthy, Fernando Silva de Moura, Juan Pablo Agnelli, and Melody Alsaker.

CUQIpy: A new Python platform for computational uncertainty quantification in inverse problems

Nicolai A. B. Riis (a), Amal M. A. Alghamdi (a), Felipe Uribe (b), Silja L. Christensen (a), Babak M. Afkham (a), Per Christian Hansen (a) and Jakob S. Jørgensen (a)

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Abstract

In this talk we present CUQIpy (pronounced "cookie pie") – a new computational modelling environment in Python that uses uncertainty quantification (UQ) to access and quantify the uncertainties in solutions to inverse problems. The overall goal of the software package is to allow both expert and non-expert (without deep knowledge of statistics and UQ) users to perform UQ related analysis of their inverse problem. To achieve this goal the package utilizes state-of-the-art tools and methods in statistics and scientific computing specifically tuned to the ill-posed and often large-scale nature of inverse problems to make UQ feasible.

We showcase the software on problems relevant to imaging science such as computed tomography and partial differential equation-based inverse problems.

CUQIpy is developed as part of the CUQI project at the Technical University of Denmark (DTU) and is available at `https://github.com/CUQI-DTU/CUQIpy`.

Optimal Experimental Design in Chromatography

Jose Rodrigo, Rojo Garcia (a), Tapio, Helin (b),
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(c): School of Engineering Science, Lappeenranta University of Technology, Lahti, Finland.

Abstract

We study Bayesian optimal experimental design in a Chromatography process. The mathematical model is governed by a non-linear hyperbolic PDE which depend of 4 unknown parameters and the 2 design variables: the time injection and the amount of concentration. The quality in the inversion process depends of the selection of the design variables and the number of nodes for measurements. The algorithms used for estimate the utility function are based in Monte Carlo estimations, and a surrogate model based in piecewise linear interpolation over sparse grids. The results conclude independence respect to the time injection for huge quantity of measurement nodes and periodicity for few nodes.

Utilizing variational autoencoders in the Bayesian inverse problem of photoacoustic tomography

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Abstract

Photoacoustic tomography (PAT) is a biomedical imaging modality based on the photoacoustic effect. In PAT, the imaged target is illuminated with a short pulse of light. Absorption of light creates localized areas of thermal expansion, resulting an increase of pressure. This initial pressure distribution relaxes as ultrasound waves that are measured on the boundary of the target. In the inverse problem of PAT, the initial pressure distribution is estimated from a set of measured ultrasound data.

In PAT, machine learning has been applied to a wide range of problems such as pre- and post processing, and partially or fully solving the inverse problem. However, as with conventional reconstruction approaches, most of the machine learning based approaches provide images of the underlying initial pressure distribution but do not offer insight in quantifying the reliability of the solution. In this work, we study a machine learning based approach to the Bayesian inverse problem of PAT [1]. The approach is based on the variational autoencoder (VAE) and the recently proposed extension to the VAE called the uncertainty quantification VAE (UQ-VAE) [2]. The proposed method provides photoacoustic images together with estimates of their reliability taking into account measurement noise, forward model and prior distribution. The approach is studied using 2D simulations and the results are compared to the solution of the Bayesian inverse problem of PAT.

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[2] H. Goh, S. Sheriffdeen, J. Wittmer, and T. Bui-Thanh, Solving bayesian inverse problems via variational autoencoders, *Proceedings of Machine Learning Research*, 145:386–425, 2021.

Uncertainty Quantification for Magnetic Resonance Electrical Impedance Tomography

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Abstract

MREIT is a promising approach for conductivity imaging. Such methods combine Electrical Impedance Tomography (EIT) and Magnetic Resonance Imaging (MRI). A magnetic resonance scanner measures the magnetic field induced by the externally applied current field from EIT. This provides additional interior information in the form of magnetic field data $\mathbf{B}$. Knowing the electrical conductivity distribution σ is vital in some medical applications, e.g. non-invasive brain stimulation where small currents are applied through electrodes to stimulate brain activity in certain regions. When performing conductivity imaging it might be that an unknown inclusion is detected. How certain can we be that the inclusion is actually there and not just a reconstruction artifact from the algorithm? How can we quantify this uncertainty mathematically?

To answer these questions one may use Bayesian inversion theory which casts the inverse problem as one of statistical inference. All quantities of interest are modeled as random variables. We parameterize the conductivity distribution by σ_p where p are unknown parameters. Then the forward problem in MREIT can be modeled as an inhomogeneous elliptic partial differential equation in three dimensions. The Bayesian solution to the inverse problem is a probability distribution called the *posterior*, which we use to infer information about p given magnetic field data. The posterior is intractable in most cases. Since we want to reconstruct the conductivity distribution σ and quantify the standard deviation of σ in the images, it amounts to characterizing the posterior by estimating its moments and statistics. Doing this for our intractable posterior is the main topic of interest.

Subaperture-based Digital Aberration Correction for Optical Coherence Tomography

Ekaterina Sherina (a), Simon Hubmer (b)

(a): Faculty of Mathematics, University of Vienna, Austria (b): Johann Radon Institute Linz, Austria

Abstract

In this talk, we consider subaperture-based approaches for the digital aberration correction (DAC) of optical coherence tomography (OCT) images. In particular, we introduce a mathematical framework for describing this class of approaches, which results in a new understanding of the previously introduced subaperture-correlation method. Furthermore, based on the insight gained by this mathematical description, we present a novel DAC approach requiring only minimal statistical assumptions on the spectral phase of the scanned object. Finally, we demonstrate the applicability of our novel DAC method via numerical examples based on both simulated and experimental OCT data.

Towards perception-aware image restoration

Samuli Siltanen

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Abstract

The goal is to measure the quality of image enhancement methods, such as deblurring, in terms of how well a person can perform a specific image-based task. One possible way to study this is to concentrate on the Non-Selective Pathway (NSP) of the human visual system [2]. It is known to quickly grasp the "gist" of an image; for example, whether the image shows a natural view or an urban environment. Are there differences in the performance of various inversion methods in terms of how well a human viewer can extract the gist from a reconstructed image? If so, perhaps sparsity-promoting inversion using Gabor-patch like building blocks performs well. A new fractal imaging method is proposed for this, based on Bayesian inversion formulated with the dual-tree complex wavelet transform [1].

References

[1] Kekkonen H, Lassas M, Saksman E and Siltanen S, *Random tree Besov priors - Towards fractal imaging.* To appear in Inverse problems and imaging.

[2] Wolfe JM, *Guided Search 6.0: An updated model of visual search. Psychonomic Bulletin & Review*, 28(4):1060–92., 2021.

Bilevel optimization with single-step inner methods

Ensio, Suonperä (a), Tuomo, Valkonen (b)

(a): Department of Mathematics and Statistics, University of Helsinki, Finland, (b): ModeMat, Escuela Politécnica Nacional, Quito, Ecuador *and* Department of Mathematics and Statistics, University of Helsinki, Finland

Abstract

We propose a new approach to solving bilevel optimization problems, intermediate between solving full-system optimality conditions with a Newton-type approach, and treating the inner problem as an implicit function. The overall idea is to solve the full-system optimality conditions, but to precondition them to alternate between taking steps of simple conventional methods for the inner problem, the adjoint equation, and the outer problem. We prove the convergence of the approach for combinations of gradient descent and forward-backward splitting with exact and inexact solution of the adjoint equation. We demonstrate good performance on learning the regularization parameter for anisotropic total variation image denoising, and the convolution kernel for image deconvolution.

Point source localisation—Lasso on measures: how to do it

Tuomo Valkonen

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Abstract

Locating stars in the sky or biomarkers in cells are examples of point source localisation problems, or superresolution beyond the diffraction limit. Such problems are generally modelled as Radon-norm regularised inverse problems on measures. Their solution demands optimisation methods in spaces of measures, which are much less developed than optimisation methods in Hilbert spaces. Most numerical algorithms for point source localisation are based on the Frank–Wolfe conditional gradient method, for which ad hoc convergence theories have been developed. In this work, we look into ways to extend other conventional optimisation methods to spaces of measures, starting from generally applicable convergence theories and first principles.

On the inference of hidden Markov models with weakly informative observations

Matti, Vihola (a)

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Abstract

Particle Markov chain Monte Carlo methods (PMCMC) [1] allow for practical Bayesian inference with a general class of dynamic models: the so-called hidden Markov models (a.k.a. general non-linear/non-Gaussian state space models). The PMCMC methods rely on combination of Markov chain Monte Carlo and particle filters. We discuss certain challenges that arise with models having 'weakly informative' observations, and present improved methods that mitigate/overcome such challenges [2,3]. The weakly informative setting arises with time-discretised path-integral models, such as a stochastic differential equation whose law is modulated by 'penalising' its paths by state-dependent (non-homogeneous and irregular) potentials.

The talk is basd on joint works with Nicolas Chopin (ENSAE), Santeri Karppinen (Jyväskylä), Sumeetpal S. Singh (Cambridge) and Tomás Soto (LUT).

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Dynamic Magnetic Particle Imaging: Accurate reconstructions by simultaneous motion estimation

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Abstract

Magnetic Particle Imaging (MPI) is a relatively new pre-clinical tracer-based tomographic imaging method. It is non-invasive and doesn't use ionizing radiation, moreover it has high spatial and temporal resolution. Potential applications include dynamic imaging tasks as blood flow imaging and instrument tracking during interventions. However, the image reconstruction task poses a severely ill-posed inverse problem even for static tracer concentrations and we face an even more challenging problem in case of dynamic concentrations.

In this talk, we propose to solve the image reconstruction task jointly with motion estimation in between the time frames, as both processes endorse each other and motion estimates are of interest in many dynamic applications. We use different motion models depending on the specific application and start from a fairly general variational problem formulation. The problem is solved by primal-dual splitting using stochastic algorithms, multi-scale approaches and image warping. We present numerical results on simulated as well as measured data.