Uncertainty Quantification for Magnetic Resonance Electrical Impedance Tomography

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Abstract

MREIT is a promising approach for conductivity imaging. Such methods combine Electrical Impedance Tomography (EIT) and Magnetic Resonance Imaging (MRI). A magnetic resonance scanner measures the magnetic field induced by the externally applied current field from EIT. This provides additional interior information in the form of magnetic field data $\mathbf{B}$. Knowing the electrical conductivity distribution σ is vital in some medical applications, e.g. non-invasive brain stimulation where small currents are applied through electrodes to stimulate brain activity in certain regions. When performing conductivity imaging it might be that an unknown inclusion is detected. How certain can we be that the inclusion is actually there and not just a reconstruction artifact from the algorithm? How can we quantify this uncertainty mathematically?

To answer these questions one may use Bayesian inversion theory which casts the inverse problem as one of statistical inference. All quantities of interest are modeled as random variables. We parameterize the conductivity distribution by σ_p where p are unknown parameters. Then the forward problem in MREIT can be modeled as an inhomogeneous elliptic partial differential equation in three dimensions. The Bayesian solution to the inverse problem is a probability distribution called the *posterior*, which we use to infer information about p given magnetic field data. The posterior is intractable in most cases. Since we want to reconstruct the conductivity distribution σ and quantify the standard deviation of σ in the images, it amounts to characterizing the posterior by estimating its moments and statistics. Doing this for our intractable posterior is the main topic of interest.