Injective Machine Learning Architectures for Inverse Problems

Michael Puthawala

Department of Mathematics and Statistics, South Dakota State University, Brookings, SD, USA

Abstract

Machine Learning has emerged as a powerful and diverse tool for solving a range of problems in science and applied mathematics. In the context of inverse problems, machine learning has, for example, been used in place of an inverse solver or as a regularizer to an existing method. In this talk we investigate the question of when neural network architectures are themselves injective. We find that networks formed by combining injective ReLU layers with bijective flow networks, we can build networks that are provably end-to-end injective. Additionally, we find that such networks come with a bevy of built-in benefits that make them well-suited for application. These include inverse stability which enables Bayesian uncertainty quantification, guarantees on universality, a novel layerwise invertibility result. Finally, we conclude with a new application where such networks are combined with a coordinate projection to produce a network that is a universal approximator of maps between compact smooth submanifolds that are locally bilipschitz, but can globally be quite complex.